

# The influence of asymmetry on a magnetized proto-neutron star.

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Using the Relativistic Mean Field Theory (RMF) it is shown that different proton fraction which is directly connected with the neutron excess and with the asymmetry of the system affects proto-neutron stars parameters and changes their composition. The obtained form of the equation of state allows to construct the mass-radius relations and shows that the increasing asymmetry creates more compact stars. The inclusion of  $\delta$  meson together with nonlinear vector meson interaction terms and magnetic field make this effect even stronger.

## Introduction

Properties of dense matter in strong magnetic field has been the subject of investigations in astrophysics of white dwarfs, proto-neutron and neutron stars. Observations of pulsars suggest large surface fields of the order of  $10^{14} \text{ G}$  [1]. This very high value indicates the existence of even stronger interior magnetic fields. In fact the virial theorem suggests that interior magnetic fields can reach the value of the order of  $10^{18} \text{ G}$ . The influence of this extraordinary high magnetic field on neutron star matter properties are of particular importance after the discovery of magnetars [1]. Their observations suggest that in the case of these objects we are dealing with young neutron stars with extremely strong surface magnetic fields  $\sim 10^{15} \text{ G}$  which in turn give an interior field of the order of  $10^{18} \text{ G}$ . Also properties of proto-neutron stars, under the influence of magnetic field should be examined. Assumptions concerning composition and the equation of state of hot, lepton rich matter have been studied by many authors (Strobel et al. 1999, Bombaci et al. 1995, Takatsuka 1995, Mańka et al. 2001) [2,3,4,5]. Matter inside a proto-neutron star is highly degenerate and chemical potentials of its constituents are a few hundreds of  $\text{MeV}$ . Like in a neutron star the strength of magnetic field of a proto-neutron star changes from  $10^8 \text{ G}$  at the surface up to  $10^{18} \text{ G}$  in the center [6]. A proto-neutron star [7,8,9] is a result of a supernova explosion which forms low entropy core with trapped neutrinos. The core is surrounded by a low density, high entropy mantle. The construction of a proto-neutron star model is based on various realistic equations of state and gives a general picture of proto-neutron star interiors. The more complete and realistic description of a proto-neutron star requires taking into consideration effects of finite temperature and nonzero magnetic fields. Using the relativistic mean-field theory approach the adequate form of the equation of state enlarged by contributions coming from magnetic field

relativistic mean field approach (RMF) and to study the influence of proton fraction on proto-neutron stars parameters. The variable proton fraction together with the inclusion of  $\delta$  meson and nonlinear vector meson interactions alter the proto-neutron stars chemical composition. This in turn affects the properties of the star making the proto-neutron star more compact.

## The generalized Relativistic Mean Field approach

The Lagrangian function for the system can be written as a sum of a baryonic part  $\mathcal{L}_B$  which includes baryon-meson interaction terms, mesonic part  $\mathcal{L}_M$  containing additional interactions between mesons which mathematically express themselves as supplementary, nonlinear terms in the Lagrangian function, the leptonic part  $\mathcal{L}_L$ , the Lagrangian density function of the QED theory  $\mathcal{L}_{QED}$  and the gravitational term  $\mathcal{L}_G$

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_L + \mathcal{L}_M + \mathcal{L}_G + \mathcal{L}_{QED}. \quad (1)$$

The final form of the Lagrangian function is given by

$$\begin{aligned} \mathcal{L} = & i\bar{\psi}\gamma^\mu D_\mu\psi - \bar{\psi}(M_N - g_\sigma\varphi - I_{3N}g_\delta\tau^a\delta^a)\psi - \kappa_N\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi \\ & - \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 - \frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} - \frac{1}{2}M_\rho^2\rho_\mu^a\rho^{a\mu} + \frac{1}{2}\partial_\mu\delta^a\partial^\mu\delta^a - \frac{1}{2}M_\delta^2\delta^a\delta^a \\ & + i\sum_{f=1}^2\bar{L}_f\gamma^\mu\widetilde{D}_\mu L_f - \sum_{f=1}^2 g_f(\bar{L}_f H e_{Rf} + h.c) - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - U(\varphi) \\ & - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{2}M_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4!}g_\rho^4\zeta(b_\mu^a b^{a\mu})^2 \\ & + (g_\rho g_\omega)^2\Lambda_v b_\mu^a b^{a\mu}\omega_\mu\omega^\mu + (g_\rho g_s)^2\Lambda_4 b_\mu^a b^{a\mu}\varphi^2 \end{aligned} \quad (2)$$

with the covariant derivatives  $D_\mu$  and  $\widetilde{D}_\mu$  defined as

$$D_\mu = \partial_\mu + \frac{1}{2}ig_\rho\rho_\mu^a\sigma^a + ig_\omega\omega_\mu + iQA_\mu, \quad \widetilde{D}_\mu = \partial_\mu + iQA_\mu. \quad (3)$$

In the case of nucleons  $Q$  takes the value  $e$  for protons ( $e$  is the electron charge) and 0 for neutrons ( $Q = e, 0$ ). In the leptonic sector ( $Q = -e, 0$ ) where  $-e$  is given for electrons and muons and 0 for neutrinos.  $H$  is Higgs field and this field has only the residual form

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Table 1a

The parameter sets of the model [14,15].

	$g_\rho$	$g_\delta$	$g_\sigma$	$g_\omega$	$M_\sigma$ (MeV)
$TM1$	9.264	0	10.0289	12.6139	511.12
$TM1 + nonl.$	10.875	3.5	10.0289	12.6139	511.12
$GM3$	8.5417	0	7.1857	8.7041	450.00

where the value  $v = 250 \text{ MeV}$  comes from the electroweak interaction scale. In a strong magnetic field, contributions coming from the anomalous magnetic moments of protons and neutrons ( $\kappa_N$ ,  $N = \{p, n\}$ ) have to be also considered. The anomalous magnetic moments introduced via the minimal coupling of nucleons to the electromagnetic field tensor can be represented phenomenologically by interactions of the type  $\kappa_N \bar{\psi} \sigma_{\mu\nu} F^{\mu\nu} \psi$ , where  $\sigma_{\mu\nu} = \frac{i}{2} \{\gamma_\mu, \gamma_\nu\}$  [14]. Considering both the physical conditions and chemical equilibrium which are likely to be satisfied in a proto-neutron star one can construct a proto-neutron star model with a temperature  $T$  equals  $20 \text{ MeV}$  and magnetic field strength  $B \sim 10^3 B_c^e$  where  $B_c^e$  is the value of the critical magnetic field for an electron  $B_c^e = m_e^2 / |e| = 4.414 \times 10^{13} \text{ G}$ . The model examined in this paper represents the one with moderate value of magnetic field. In this case the contributions to the equation of state coming from anomalous magnetic moments are small ( $\sim 1 \text{ MeV}$ ) in comparison with the energy of magnetic field. Thus this relatively small value of magnetic field allows to neglect the anomalous magnetic moments unlike to the case describe in the paper by Prakash et al. [14] where authors consider the case of extremely strong magnetic field of the order of  $10^{18} \text{ G}$ . Realistic proto-neutron star models describe electrically neutral, hot, high density matter being in  $\beta$  equilibrium. The last condition implies the presence of leptons which is expressed by adding the Lagrangian of leptons to the Lagrangian function. The fermion fields which are included in this model consists of neutrons, protons, electrons, muons and neutrinos

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}, \quad L_1 = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad L_2 = \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad e_{Rf} = (e_R^-, \mu_R^-). \quad (4)$$

As a proto-neutron star matter is of sizeable asymmetry the additional  $\delta$  meson has been included and the meson sector is composed of isoscalar (scalar  $\sigma$  and vector  $\omega$ ) and isovector (scalar  $\delta$  and vector  $\rho$ ) mesons. The potential function  $U(\varphi) = \frac{1}{2} M_\sigma^2 \varphi^2 + \frac{1}{3} g_2 \varphi^3 + \frac{1}{4} g_3 \varphi^4$  has a very well known form introduced by Boguta and Bodmer [16]. Nucleon masses are denoted by  $M_N$  ( $N=p,n$ ) whereas  $M_\omega$ ,  $M_\rho$ ,  $M_\sigma$  and  $M_\delta$  are masses assigned to the meson fields. They are taken at their experimentally values:  $M_\omega = 783 \text{ MeV}$ ,  $M_\rho = 770 \text{ MeV}$

Table 1b

The self-interacting coupling constants [14,15].

	$g_2$ (MeV)	$g_3$	$c_3$	$\Lambda_v$	$\Lambda_4$	$\zeta$
<i>TM1</i>	1427.18	0.6183	71.3075	—	—	—
<i>TM1 + nonl.</i>	1427.18	0.6183	71.3075	0.008	0.001	0.5
<i>GM3</i>	3016.52	-6.4546	0	—	—	—

and  $M_\delta = 980 \text{ MeV}$ . The nonlinear vector meson interaction terms have been added in order to give a detailed account of a matter inside a proto-neutron star. This has been done through the construction of adequate form of the equation of state. To determine the equation of state at finite temperature and nonvanishing magnetic field some parameterizations have been used. The first one is known as *TM1* parameter set [15], the second one noted as *TM1+nonl.* includes nonlinear terms in the meson sector. The third is the *GM3* parameter set [14]. The parameters entering the Lagrangian function are the coupling constants  $g_\rho, g_\delta, g_\sigma, g_\omega$  for meson fields and self-interacting coupling constants  $g_2, g_3, c_3, \Lambda_v, \Lambda_4$  and  $\zeta$ . All these parameters have been chosen to reproduce properties of symmetric nuclear matter at saturation and they are collected in Tables 1a and 1b. In the original *GM3* and *TM1* approaches neutrinos are not included since these parameterizations describe a neutron star matter. In this case neutrinos leave the star unhindered. In order to construct the neutron star model through the entire density span the addition of the equations of state, characteristic for the inner and outer core, relevant for lower densities is necessary. In the result the composite equation of state for which the *TM1* or *GM3* parameter groups describing the neutron star core are supplemented with the Negele-Voutherin (NV) + Bonn describing inner crust can be used. The equilibrium properties of nuclear matter which are known with reasonable precision are: the nuclear saturation density  $n_S$ , the binding energy  $E_b$ , the incompressibility  $K_0$ , the symmetry energy  $a_{sym}$  and the effective nucleon mass. Nuclear matter properties calculated for given parameter sets (Tables 1a and 1b) are summarized in Table 2. The presence of additional nonlinear  $\rho$  meson interaction terms and  $\delta$  meson which carries isospin contribute to the symmetry energy. Thus the value of the parameter  $g_\rho$  has to be redefined for the *TM1 + nonl.* parameterization. Parameters  $g_\rho, g_\delta, \Lambda_v, \Lambda_4$  and  $\zeta$  are not chosen arbitrary they have to be given in a specified form adequate to reproduce the value of the symmetry energy  $a_{sym}$ . The vector potential for the electromagnetic stress tensor  $F_{\mu\nu}$  is given by  $A_\mu = \{A_0 = 0, A_i\}$  where

$$A_i = -\frac{1}{2}\varepsilon_{ilm}x^l B_0^m.$$

Table 2

The nuclear matter properties.

	$E_b (MeV)$	$m_{eff}/M$	$n_S (fm^{-3})$	$K_0 (MeV)$	$a_{sym} (MeV)$
$TM1$	-16.3	0.658	0.1455	281.99	36.82
$TM1 + nonl.$	-16.3	0.664	0.1455	281.99	32.22
$GM3$	-16.3	0.78	0.153	240	32.5

The symmetry in which uniform magnetic field  $\mathbf{B}_0$  lies along the z-axis has been chosen  $B_0^m = (0, 0, B_z)$ . Considering the influence of an external magnetic field on proto-neutron star constituents one can start with determining the dispersion relations for them. The dispersion relation for an electron in a magnetic field is

$$\varepsilon_e = \sqrt{p_z^2 + m_e^2 + 2neB_z},$$

where  $n$  is the Landau level,  $p_z$  is the electron momentum along the z-axis and  $m_e$  is the rest mass of the electron [17]. Along the field direction a particle motion is free and quasi-one-dimensional with the modified density of states which is given by a sum

$$2 \int \frac{d^3p}{(2\pi)^3} \rightarrow \sum_s \sum_{n=0}^{\infty} [2 - \delta_{n0}] \int \frac{eB_z}{(2\pi)^2} dp_z,$$

$\delta_{n0}$  denotes the Kronecker delta [17]. The spin degeneracy equals 1 for the ground ( $n = 0$ ) Landau level and 2 for  $n \geq 1$ . Similar results can be obtained for muons by replacing the electron quantities by the corresponding muon quantities. The energy dispersion relation for protons for arbitrary Landau level in a magnetic field is [14]

$$\varepsilon_{p,s} = \sqrt{p_z^2 + (\sqrt{M_{p,eff}^2 + 2nQB_z} + s\kappa_p B_z)^2} \quad (5)$$

and  $s = \pm 1$  being "spin" number. For moderate values of magnetic fields strength this equation reduces to the following one

$$\varepsilon_p \sim \sqrt{p_z^2 + M_{p,eff}^2 + 2neB_z}.$$

A neutron can interact with an external electromagnetic field by means of the Pauli non-minimal coupling. Then the energy dispersion relation for neutrons

in a magnetic field is given by

$$\varepsilon_{n,s} = \sqrt{p_{\parallel}^2 + (\sqrt{M_{n,eff}^2 + p_{\perp}^2} + s\kappa_n B_z)^2} \quad (6)$$

where  $p_{\parallel} = p_z$  and  $p_{\perp}$  are the components of the neutron momentum parallel and perpendicular to the magnetic field. As it is usually assumed in quantum hadrodynamics the mean field approximation is adopted and for the ground state of homogeneous infinite matter quantum fields operators are replaced by their classical expectation values. One can separate mesonic fields into classical mean field values and quantum fluctuations which are not included in the ground state and thus mesons fields are replaced by their mean values  $\sigma = \langle \varphi \rangle$ ,  $d^a = \langle \delta^a \rangle = \delta^{a,3} d_0$ ,  $w_{\mu} = \langle \omega_{\mu} \rangle = \delta_{\mu,0} w_0$  and  $r_{\mu}^a = \langle \rho_{\mu}^a \rangle = \delta^{a,3} \delta_{\mu,0} r_0$ . The Dirac equation at the mean field level for the nucleon quasi-particle has the form

$$(i\gamma^{\mu} D_{\mu} - M_{N,eff} - \kappa_N \sigma_{\mu\nu} F^{\mu\nu})\psi = 0 \quad (7)$$

with  $M_{N,eff}$  being the effective nucleon mass generated by the nucleon and scalar fields interactions and defined as

$$M_{p,eff} = M\delta_p = M - g_s\sigma - g_{\delta}d_0, \quad (8)$$

$$M_{n,eff} = M\delta_n = M - g_s\sigma + g_{\delta}d_0. \quad (9)$$

The main effect of the inclusion of  $\delta$  meson becomes evident studying properties of proto-neutron star matter especially nucleon mass splitting and the form of the equation of state. With the effective nucleon mass  $M_{N,eff}$  one can redefine the proton and neutron chemical potentials

$$\mu_p = \varepsilon_p + g_{\omega}w_0 + \frac{1}{2}g_{\rho}r_0 \quad (10)$$

$$\mu_n = \varepsilon_n + g_{\omega}w_0 - \frac{1}{2}g_{\rho}r_0 \quad (11)$$

where  $\varepsilon_p$  and  $\varepsilon_n$  is given by relations (6) and (5). The obtained effective nucleon chemical potentials allow to define conditions that are regarded as essential to uniquely determine the complete equilibrium composition of proto-neutron star matter at given density. These conditions arise from charge neutrality, baryon and lepton number conservation. The last one is strictly connected with the assumption that proto-neutron star matter is opaque to neutrinos. Neutrinos are trapped inside the matter and this through the requirement of  $\beta$  equilibrium puts certain limits on the chemical composition of a proto-neutron star matter. For the case to be considered the processes of  $\beta$ -decay and inverse  $\beta$ -decay take place [18]

$$p + e \leftrightarrow n + \nu_e. \quad (12)$$

For large enough electron Fermi energy it is energetically favorable for electrons to convert to muons

$$\mu + \nu_e \leftrightarrow e + \nu_\mu. \quad (13)$$

The chemical equilibrium established by the above processes is given by relations between chemical potentials of proto-neutron star constituents and impose constraints on them.

$$\mu_{\nu_e} = \mu_e + \mu_p - \mu_n \quad (14)$$

$$\mu_{\nu_\mu} = \mu_{\nu_e} - \mu_e + \mu_\mu. \quad (15)$$

From the equations (10) and (11) the electron neutrino chemical potential can be expressed in the following way

$$\mu_{\nu_e} = \mu_e + \varepsilon_p - \varepsilon_n + g_\rho r_0. \quad (16)$$

With the assumption that only electron neutrinos are captured in the star core ( $\mu_{\nu_\mu} = 0$ ) the following relations for electron neutrino and muon chemical potentials can be obtained

$$\begin{aligned} \mu_{\nu_e} &= \mu_e + \varepsilon_p - \varepsilon_n + g_\rho r_0 \\ \mu_\mu &= -\mu_{\nu_e} + \mu_e = -\varepsilon_p + \varepsilon_n - g_\rho r_0. \end{aligned} \quad (17)$$

Both electron neutrino and muon chemical potentials depend on nucleon asymmetry. The muon chemical potential depends on the asymmetry of the whole system through the value of the  $\rho$  meson field which is directly connected with the neutron excess and the difference of nucleon energies. Comparison between the matter with high asymmetry ( $Y_p = 0.11$ ) and the one with low asymmetry indicates that in the former case the muon chemical potential is greater than that of the later one. This translates to the lower number of muons in the proto-neutron star matter with the proton number  $Y_p$  of the order of 0.38. The increase of neutron excess is connected with the increasing number of muons. Their contribution to the pressure and energy density of the system become significant. To construct the equation of state of the system the energy-momentum tensor has to be calculated. The pressure is related to the statistical average of the trace of the spatial components of the energy-momentum tensor  $P = \frac{1}{3} \langle T_{ii} \rangle$  whereas the energy density  $\varepsilon$  equals  $\langle T_{00} \rangle$ . In general in the presence of magnetic field the pressure can be written as a sum of the isotropic part which includes contributions coming from electrons, nucleons, mesons and the electromagnetic anisotropic part  $T_{\mu\nu}^B$  is given by

$$T_{\mu\nu}^B = F_\mu^\lambda F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}.$$

In cartesian coordinates the electromagnetic part of the energy momentum tensor in the flat space-time has the form

$$T_{\mu\nu}^B = \begin{pmatrix} \frac{1}{2}B^2 & 0 & 0 & 0 \\ 0 & \frac{1}{2}B^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}B^2 & 0 \\ 0 & 0 & 0 & -\frac{1}{2}B^2 \end{pmatrix}. \quad (18)$$

On the other hand in polar coordinates  $T_{\mu\nu}^B$  has been written as follows

$$\begin{pmatrix} \frac{1}{2}B^2 & 0 & 0 & 0 \\ 0 & -\frac{1}{2}B^2 \cos 2\theta & \frac{1}{2}B^2 r \sin 2\theta & 0 \\ 0 & \frac{1}{2}B^2 r \sin 2\theta & \frac{1}{2}B^2 r^2 \cos 2\theta & 0 \\ 0 & 0 & 0 & \frac{1}{2}B^2 r^2 \sin^2 \theta \end{pmatrix}.$$

Taking the average over all directions allows to obtain the isotropic form of the energy momentum tensor relevant for the spherical symmetric configuration

$$\begin{pmatrix} \frac{1}{2}B^2 & 0 & 0 & 0 \\ 0 & \frac{1}{6}B^2 & 0 & 0 \\ 0 & 0 & \frac{1}{6}B^2 & 0 \\ 0 & 0 & 0 & \frac{1}{6}B^2 \end{pmatrix}.$$

The assumption of the positivity of the total pressure  $P$  has been made. Any negative contributions to the pressure diminish it and lead to the decrease of a star radius. The described above averaging approximates the star as an spherically symmetric object with the radius  $R$  which is smaller than the equatorial radius of an anisotropic star. (The case of an anisotropic star is calculated in details in the paper by Konno et al. [19]). The condition of the positivity of the total pressure establishes a limiting value of the density noted as  $\rho_{crit}$  and at the same time puts limit on the spherical symmetric approximation (the deformation of the star is neglected). For densities smaller than the critical one the total pressure became negative and there is no gravity compensation in this direction. The total pressure and the energy density of the system supplemented by corrections coming from magnetic field are obtained by adding contributions from mesons and fermions



$$\begin{aligned}
P &= P_f + P_{QED} - U(\sigma) + \frac{1}{2}M_\rho^2 r_0^2 - \frac{1}{2}M_\omega^2 w_0^2 - \frac{1}{2}M_\delta^2 d_0^2 + \Lambda_v g_\omega^2 g_\rho^2 r_0^2 w_0^2 + \\
&\quad + g_\sigma^2 g_\rho^2 \Lambda_4 r_0^2 \sigma^2 + \frac{1}{4}c_3 w_0^4 + \frac{1}{24}\zeta g_\rho^4 r_0^4 \\
\varepsilon &= \varepsilon_f + \varepsilon_{QED} + U(\sigma) + \frac{3}{4}c_3 w_0^4 + \frac{1}{2}M_\omega^2 w_0^2 + \frac{1}{2}M_\rho^2 r_0^2 + 3\Lambda_v g_\omega^2 g_\rho^2 r_0^2 w_0^2 + g_\sigma^2 g_\rho^2 \\
&\quad \Lambda_4 r_0^2 \sigma^2 + \frac{1}{8}\zeta g_\rho^4 r_0^4 + \frac{1}{2}M_\delta^2 d_0^2
\end{aligned}$$

where  $f = (i, n)$ ,  $i = (e, \mu, p)$  and  $\varepsilon_{QED} = \frac{1}{2}B^2$ ,  $P_{QED} = \frac{1}{6}B^2$ . The fermion pressure is defined as

$$\begin{aligned}
P_i &= \frac{\gamma_e m_i^4}{4\pi^2} \sum_s \sum_{n=0}^{\infty} [2 - \delta_{n0}] \{ I_{-1,2,+}(z_i/t_i, \varsigma_i^2 + 2\gamma_e n) \\
&\quad + I_{-1,2,-}(z_i/t_i, \varsigma_i^2 + 2\gamma_e n) \}
\end{aligned}$$

where the Fermi integral

$$I_{\lambda,\eta\pm}(u, \alpha) = \int \frac{(\alpha + x^2)^{\lambda/2} x^\eta dx}{e^{(\sqrt{\alpha+x^2} \mp u)} + 1} \quad (19)$$

was used,  $z_i = \mu_0/m_i$ ,  $t_i = k_B T_0/m_i$ ,  $u_i = z_i/t_i$ ,  $\gamma_e = B_z/B_c^e$  and  $\varsigma_i = m_{i,eff}/m_i$ . The last quantity equals 1 for electrons and muons. The critical magnetic field strength for electron equals  $B_c^e = m_e^2 / |e| = 4.414 \times 10^{13} \text{ G}$ . The fermion energy density is also defined with the use of the Fermi integral

$$\begin{aligned}
\varepsilon_i &= \frac{\gamma_e m_i^4}{4\pi^2} \sum_s \sum_{n=0}^{\infty} [2 - \delta_{n0}] \{ I_{1,0,+}(z_i/t_i, \varsigma_i^2 + 2\gamma_e n) \\
&\quad + I_{1,0,-}(z_i/t_i, \varsigma_i^2 + 2\gamma_e n) \}.
\end{aligned}$$

## Conclusion

Analyzing properties of proto-neutron star matter some features which are relevant for proto-neutron stars have to be considered. It is a necessary assumption that the matter is opaque to neutrinos. As a result neutrinos are trapped dynamically inside the matter on the diffusion time scale and in contradiction to neutron star matter the total lepton number  $Y_L = Y_e + Y_{\mu_e} + Y_{\mu}$  is kept constant. Theoretical predictions determine the value of  $Y_L$  at the onset of trapping at about 0.35-0.4. Since a proto-neutron star is formed during a supernova explosion it is safely to make an assumption about finite temperature

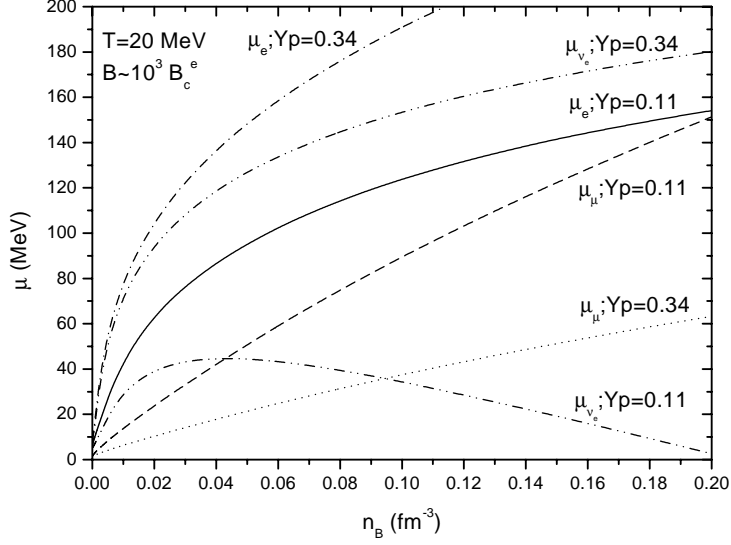


Figure 1. The lepton chemical potentials ( $e, \mu, \nu_e$ ) as a function of the baryon density.

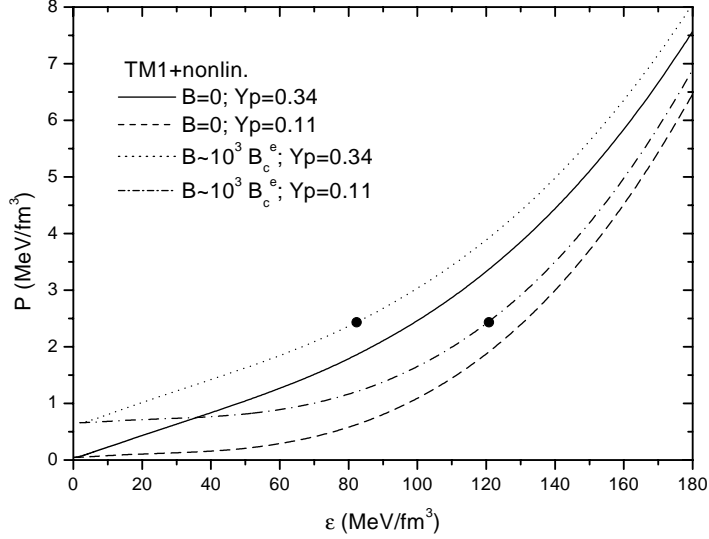


Figure 2. The equation of state for the proto-neutron star for different values of the proton concentration  $Y_p$  and strength of magnetic field. TM1 model with the nonlinear terms.

in the interior of a star. This is limited by the requirement of constant value of entropy in a proto-neutron star core. In this paper the temperature is fixed and equals 20 MeV. Thus conditions which have to be considered namely: finite temperature, lepton number conservation,  $\beta$ - equilibrium, charge neutrality and additionally nonzero magnetic field allow to construct a realistic proto-neutron star model. All of these mentioned above conditions influence medium

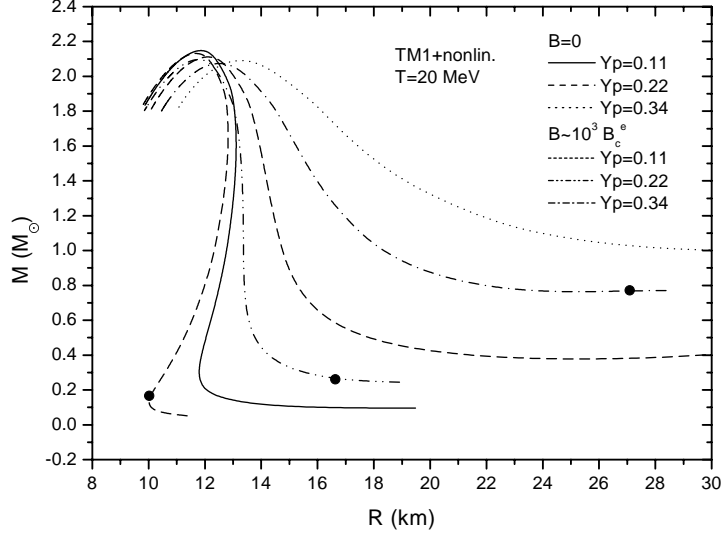


Figure 3. The mass radius relations for different proton fractions  $Y_p$  ( $T = 20, \text{ MeV}$ ,  $B \sim 10^3 B_c^e$ ,  $B_c^e = 4.414 \times 10^{13}$ ). For comparison the mass radius relation for the proto-neutron star without magnetic field ( $B = 0$ ) is also presented).

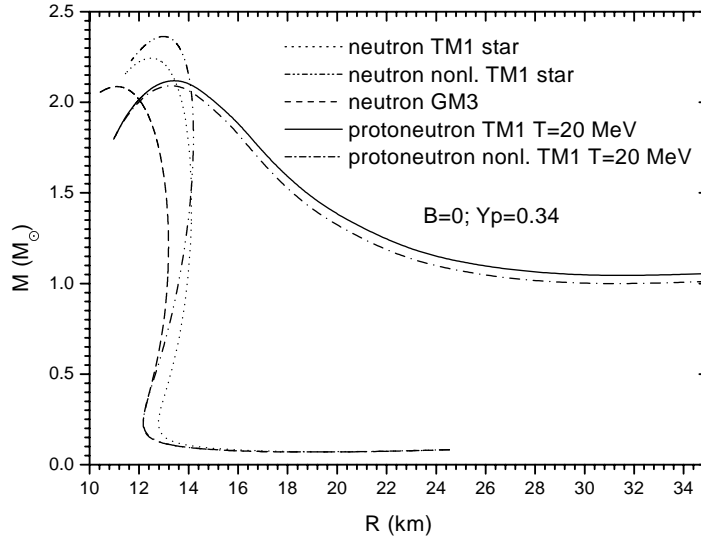


Figure 4. The mass radius relations for neutron and proto-neutron stars for different parameter sets.

properties such as nucleon effective masses, density and proton-neutron asymmetry and in consequence constrain the nucleon and lepton species inside the star core. The proton fraction  $Y_p = n_p/(n_p + n_n)$  which is determined by  $\beta$  equilibrium, for matter transparent to neutrinos (neutron star matter) takes

the value of the order of  $\sim 0.1$  whereas when neutrinos do contribute they displace the equilibrium proton fraction to the higher value  $Y_p \sim 0.3 - 0.36$ . For the purpose of comparison all calculations have been performed for decreasing value of proton fractions  $Y_p = 0.34, Y_p = 0.22, Y_p = 0.11$ , respectively. This changing proton-neutron asymmetry through relation (17) alters the lepton chemical potentials. Figure 1 displays chemical potentials of electron neutrinos, electrons and muons for two extreme values of  $Y_p$  ( $Y_p = 0.11$  and  $Y_p = 0.34$ ). They have been examined for moderate strength of magnetic field. If the proton fraction equals  $Y_p = 0.34$  (the proto-neutron star matter) electron and neutrino electron chemical potentials are increasing functions of a density. Neutrinos are trapped inside the matter and their chemical potential differs from zero (see relation 16). For the proton fraction  $Y_p = 0.11$  the matter inside the star resembles that of a neutron star which is characterized by larger value of asymmetry. Neutrinos are about to leave the system and their chemical potential vanishes  $\mu_\nu \rightarrow 0$ . This picture shows the profile of neutrino chemical potential in the core of the star for the chosen values of  $Y_p$  and for chosen parameter set (TM1+nonl.). For the star with the value of asymmetry typical for a proto-neutron star matter ( $Y_p \sim 0.38$ ) muons are of insignificant meaning. However, they become much more important for lower value of proton number. The asymmetry entered into calculations at different levels starting from the displacement of proton and neutron chemical potentials  $\mu_p \neq \mu_n$ . In a model with  $\delta$  meson exchange additionally  $M_p \neq M_n$ . There are also contributions coming from nonlinear vector meson interactions. The nuclear asymmetry diminishes the neutrino chemical potential and thus lowers neutrino density. The influence of the asymmetry is most distinctive in the high density region inside a proto-neutron star. The forms of neutrino profiles in the proto-neutron star core depend on the asymmetry of the whole system. The behavior of chemical potentials translates to the forms of the equation of state and successively to the mass-radius relations. Results are obtained for two particular cases namely for the nonmagnetic one ( $B = 0$ ) and for the value of magnetic field  $B \sim 10^3 B_c^e$ . The presence of magnetic field makes the equation of state stiffer (Fig. 2), whereas for more asymmetric matter the obtained equation of state is softer. Dots represent the limit of making the approximation of spherically symmetric star in the presence of magnetic field. For more asymmetric matter the dot is shifted towards higher densities. In Fig. 3 one can compare the influence of changing proton fraction on star parameters. The obtained sequences of mass-radius for nonmagnetic and magnetic proto-neutron stars show that the increasing asymmetry diminishes proto-neutron star radii causing that the star parameters start to resemble that of neutron stars. This effect is even stronger in the presence of magnetic field. There are relatively minor changes in the value of maximal masses caused by the variation of proton fraction. However, there is a tendency of the increase of the maximal mass for growing asymmetry. The star parameters are also sensitive to the presence of nonlinear vector meson interaction terms in the Lagrangian function (2). This is clearly visible in Fig. 4. Comparison of results obtained

for original GM3 and TM1 equations of state (for neutron stars) with that obtained for TM1+nonl. shows that the last one gives the configuration with the bigger value of maximal mass. In the range of lower densities the TM1+nonl. parameter set gives the star with the smaller radius. Thus in the case of high asymmetry nonlinearities create more compact objects. Nonlinearities alter proto-neutron star parameters insignificantly. This is caused by lower value of the asymmetry.

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